On New Approaches in Nonlinear Dynamics of Complex Systems

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SUMMARY: An approach is introduced that tries to make sense of interaction of physical processes characterized by strongly different space and especially time scales. Slow processes can be assumed as “rules” that govern fast processes, but it’s argued that some feedback must exist. Some new models are also introduced. The first is extension of K. Sneppen SOC model of solid body – vapor interface depinning. The second is model of a toy “physical” Universe with evolving “physical laws”. The last one imitates evolution of animal population with slowly evolving instincts.

Complexity as multi-scaled phenomena

Today we can see the birth of a new paradigm in nonlinear science. On our opinion it’s connected to understanding that actually complex phenomena involve a wide range of time and space scales.

At first it’s the concept of *self-organized criticality* (SOC) by *P. Bak* and *K. Chan*. Many natural systems turned out to obey power law statistics. Among them there are flashes of the sun activity, oscillations of air dampness, clouds formation, forest fires, earthquakes, forming of rivers web, work of human brain, dynamics of traffic flow, biological evolution, and many others. The power law statistics changes traditional view of rare events. If you deal with such systems as laser plasma, you can be sure that you can practically omit molecules with velocities many times grater then $\propto \sqrt{\frac{T}{m}}$, where T is temperature, and m is molecule mass, because the Maxwell velocity distribution has exponentially small “tails”. But it is not the same with SOC objects like earthquakes, when the distribution of energies looks like $\rho(E) \propto \frac{1}{E^{\alpha}}, \alpha \propto 1$. The possibility to clash with enormously strong event is not vanishing.

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The most popular and simple model of SOC phenomena is cellular automaton *pile of sand*. Imagine that you are God and you pour the sand on the sinful Earth grain by grain. A kind of pile emerges downstairs and while the slope of the pile is small nothing interesting happens. But when the slope reaches some critical value an avalanche occurs. The distribution of avalanche sizes and avalanche duration obey just the power law statistics.

Presence of a wide range of time and distance scales is typical for another simple models, for class IV cellular automata (like *J. Conway*'s *Game of life*) for example. Namely, it was found that when stationary configuration in the *Game of life* is randomly perturbed avalanches occur like those of sand of pile, which also obey the power law statistics

In Keldysh Institute of Applied Mathematics in Moscow an interesting extension of cellular automata – like SOC models was proposed [1)]. This new model is a variant of *K.Sneppen* cellular automaton describing the solid body – vapor interface depinning. The model demonstrates a new phenomenon of *soft universality,* which means that you can change a single index (without affecting other indices) of your power law distributions of model parameters by simply adjusting the only parameter. This shows that the class of objects that can demonstrate SOC behavior can be even wider then considered thus far and this class can be *continuous* in a sense.

The next step: interaction of slow and fast processes

So we conclude that wide range of time and distance scales is serious sign of complex system. Now we're going to propose some ideas that, on our opinion, can turn formulated properties from passive establishing of facts to active analyzing of real systems. We assume that *interaction* between slow and fast processes to be responsible for the nature of complexity.

Actually some researchers have already come very close to such assumption. V.I.Vernadsky mentioned the hierarchy of times in biological phenomena. By *V.I. Vernadsky*, there are three main time scales: individual lifetime, population lifetime, and species lifetime. *N.N. Moiseyev* proposed a concept of *universal evolutionism*. According to this concept the laws of Nature may be a result of a random choice at previous level of organization of matter. Then we should mention investigations accomplished by *G.I. Marchuk* in the field of weather and climate dynamics. It was found that one couldn't predict weather changes without taking into account slow changes of climate. That means just the assumed interaction of differently scaled processes.

The mechanism of interaction between slow and fast processes isn't actually understood thus far. Meanwhile it must be the most interesting. When solved, this mechanism may supply understanding of what *information* is. Imagine a living cell. The hierarchy of times does actually exist here: one can detect the lifetime of a single protein molecule, the lifetime of a whole cell, the lifetime of genetic code (i.e. lifetime of DNA molecule or it's parts), and at last, the lifetime of three-nucleotide code, which is used in the cell to transfer DNA records to protein structure. The first mentioned time might scale minutes, while the last one scales millions of years. One can easily see here, that coexistence of time scales is somehow connected to information storage.

We think interaction means double-sided movement. It's clear how slow processes like climate evolution can influence upon fast phenomena like weather. It was proposed by *S. Wolfram* to investigate cellular automata with two layers [2)], when the *slow* layer determines evolution rules to the *fast* layer. But what does it mean when fast dynamics acts upon slow one? How can the weather changes influence upon climate?

W.B. Arthur from *Santa Fe Institute* proposed the concept of positive feedback in economy. This feedback means that if some technology is used already this enlarges technology's chances to win the markets. So the final choice of dominating technology may be not a result of its advantages, but a random choice, just like in *N.N. Moiseyev*'s concept of universal evolutionism. But contrary to physics, the nature of feedback is clear in economy: consumers of high-tech production naturally prefer well-known devices, because they can easily get instructions on device using and well-known products seem to be more reliable. *W.B.Arthur* cites a bright example of positive feedback. The "QWERTY" keyboard was invented by *C. Scholes* in 1873 in the USA just to make typing maximally not convenient, because old typing machines jammed at fast typing. There's no more any problem with fast typing, nevertheless feedback still works and the system still remembers the initial choice. This example gives a clear sight of different time scales (single consumer choice – market choice) interaction, information storage, and connection of these phenomena.

Researchers dealing with SOC also came recently to understanding that feedback from fast dynamics to slow one must exist: "...a feedback mechanism must operate which, from perspective of usual critical phenomena, describes the action of the order parameter onto control parameter and attracts dynamics to a critical state" [3]. The concept of evolutionary stable strategy (ESS) developed by *J.Maynard Smith* [4] also assumes feedback from animal's behavior to their genes, i.e. from fast processes to slow.

Simple models

Here we introduce some simple mathematical models developed in M.V. Keldysh Institute, which make attempt to illustrate formulated principles. We consider here slow processes as *rules* or *laws* that determine fast behavior of a system. On the other hand we assume some kind of hypothetical feedback, that acts from fast dynamics onto rules (Fig. 1).

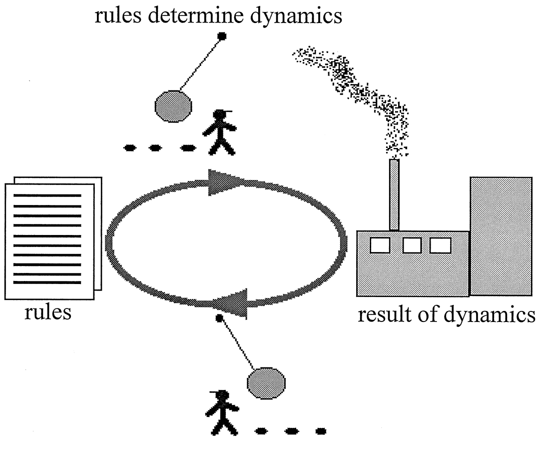

Fig. 1. Main idea of co-evolution of fast and slow parameters

First a cellular automaton-based model was developed [5], which simulates an evolution of two-dimensional cellular universe. It's assumed in the model, that initially each site of the automaton has it's own set of rules, just like a living cell has it's own set of DNA molecules. Each single rule in each cell has a finite lifetime (information storage time) and randomly changes (mutates) at the end of its life. On the other hand a single rule may be encouraged by an additional "life" of the same duration, if the rule *works*, i.e. if a certain combination of site state and site's neighbors states is realized in a moment. So a natural selection of rules goes on, when only necessary rules survive. This co-evolution of "physical states" and "physical laws" lead to emergence of stable (eternal) local rules (not the same for the whole cellular universe) and stable and oscillating structures for physical states of sites over the "layer" of these rules (Fig. 2).

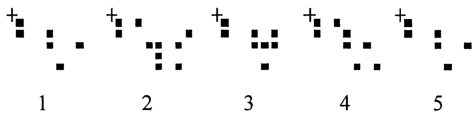

Fig. 2. 4-cycle on a substrate of constant rules

So we have situation when interaction of fast dynamics (single time step of duration 1) with slow dynamics (lifetime of a single rule of duration $\propto$ 10) leads to emergence of the third time scale (∞) and storage of information.

Another model makes a new look at simple logistic map:

$$x_{n+1} = a\,x_n(1-x_n) \qquad (1)$$

which is sometimes assumed to simulate some aspects of animal population dynamics. We assumed parameter a to be not a externally given constant, but also a dynamical variable and supposed

$$\begin{cases} x_{n+1} = a_n\, x_n\,(1-x_n) \\ a_{n+1} = a_n(1-\delta) + \varepsilon\, f(a_n)\, g(\bar{x}_{n,N}) \end{cases}, \quad \bar{x}_{n,T} = \frac{1}{N}\sum_{k=n-N+1}^{n} x_k, \quad \delta, \varepsilon << 1 \qquad (2)$$

where $f(a)$ and $g(x)$ are some nonlinear functions. We call this map as a "clock with winding up". Imagine that parameter a, which characterizes intensity of individual interactions in a population, is determined by some kind of oscillator (maybe biochemical) in an averaged single animal in the population. This oscillator may be regulated genetically, and so its energy slowly decreases, which is characterized by small parameter δ. On the other side this biochemical clock is winded up somehow through individual interactions of animals, which is expressed by second member in mapping for a_{n+1}. Intensity of this winding up is characterized by small parameter ε.

The model gave very interesting output even for the simplest case $f(a) = 1 = const,\ g(x) = x$ (Fig. 3). Note that for about $\approx$ 400 steps fast variable x_n has just it's quasi-stationary value

$x_n \approx \bar{x}(a_n) = 1 - \frac{1}{a_n}$, as if slow variable a_n is constant. But the surprise is that this stable point for x_n becomes unstable for $a > 3$ if $a = const$. Meanwhile in our case $a_n > 3$ since already ≈ 250 time steps. The matter is co-evolution of fast and slow variables, which prolongs the lifetime of unstable point.

Maybe that's *why* some catastrophic events (like the crash of the Soviet Union, for example) seem unexpected for us, though not the crash but *late* and *delayed* crash must actually surprise? Like SOC models, this one shows, that many complex systems may exist (perhaps even may *only* exist) in critical state, a single step from destroying.

The model (2) can also demonstrate many other scenarios of evolution [6]. For example one can obtain looking like history evolutions with flourishing and regress of "civilizations", vibration regimes (both regular and chaotic).

How to understand the most complex phenomena

We believe that investigations of complex systems including joint consideration of dynamics of "rules" and "physical values", i.e. of slow and fast variables, may supply much more understanding of their dynamics. It seems that polymer phenomena is very attractive field for formulated approach. We should even prompt to researchers: *just look for feedback from fast processes to slow, and you find many surprising phenomena.*

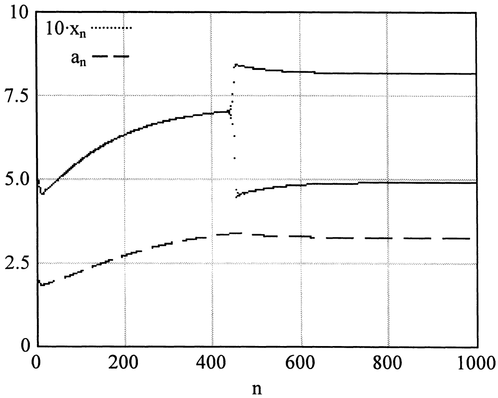

Fig. 3. Co-evolution of fast variable x and slow variable a ("rules")

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